

Particle Horizon and Warped Phenomenology

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Abstract

Giant resonances of gravity Kaluza-Klein modes (with tensor couplings) in high energy collisions are expected in the Randall-Sundrum orbifold model that incorporates a plausible solution to the hierarchy problem. When the model is extended to incorporate an exponentially small 4-D cosmological constant, the KK spectrum becomes continuous, even in the compactified case. This is due to the presence of a particle horizon, which provides a way to evade Weinberg's argument of the need of fine-tuning to get a very small cosmological constant.

I. INTRODUCTION

Recently Randall and Sundrum used warped geometry to propose a plausible solution to the hierarchy problem [1]. In their S^1/\mathbb{Z}_2 orbifold model, the two branes sit at the two fixed end points. As a result of the compactification, the discrete gravity Kaluza-Klein (KK) modes have relatively strong (compared to the graviton) couplings to matter fields on the visible brane. This implies giant resonances in high energy collisions [1,2].

The warped geometry idea was recently used to provide a plausible solution to the cosmological constant problem [4]. More recently, in Ref [5], this scenario was further extended to incorporate the hierarchy solution of the original proposal [1]. In this paper, we point out that the phenomenology changes drastically in this scenario, even though the hierarchy solution in this new scenario is very similar to that of the S^1/\mathbb{Z}_2 orbifold model [1]. Furthermore, this change, from a discrete to a continuous KK spectrum, happens irrespective of whether the extra dimension is compactified or not. It has to do with the presence of the particle horizon that invariably appears in any scenario of the type of Ref [4,5] that naturally incorporates an exponentially small 4-D cosmological constant. In some sense, the phenomenology of the continuous gravity KK spectrum in this scenario (compactified or not) is quite similar to that on the probe brane [3] in the presence of the Planck brane in the uncompactified model also proposed by Randall and Sundrum [2].

To be explicit, let us first discuss the specific two brane compactified model presented in Ref [5]. (We shall discuss the general situation in a moment.) Consider two parallel 3-branes with brane tensions $\sigma_0 > \sigma_1 > 0$ sitting in a compactified 5th (i.e., y) dimension, with circumference $L_2 - L_0$. The σ_0 (Planck, hidden) brane sits at $L_0 = 0$ and the σ_1 (TeV, visible) brane sits at L_1 . All matter fields of the standard model of strong and electroweak interactions are confined on the visible brane. Since L_2 is identified with $L_0 = 0$, this means the branes are separated by L_1 on one side and by $L_2 - L_1$ on the other side. Without loss of generality, let $L_2 - L_1 > L_1$. Using the metric ansatz

$$ds^2 = dy^2 + A(y)[-dt^2 + \exp(2Ht)\delta_{ij}dx^i dx^j]. \quad (1)$$

where, in the absence of matter density, the Hubble constant H is truly a constant. The Einstein equation yields the general solution [10]

$$A(y) = \frac{H^2 \sinh^2[k_i(y - y_i)]}{k_i^2}, \quad (2)$$

in which k_i ($k_i^2 = \kappa^2 |\Lambda_i|/6$ where κ^2 is the 5-D gravitational coupling and Λ_i are the bulk cosmological constants) and y_i are integration constants to be fixed by the boundary conditions at the branes. The warp factor $A(y)$ is schematically shown in Figure 1.

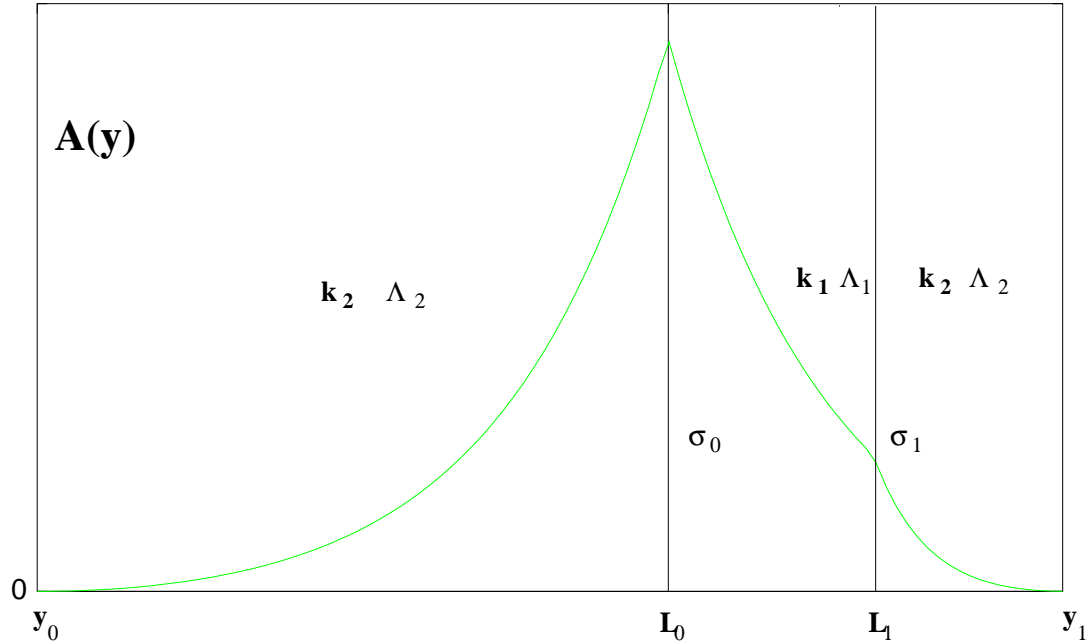


FIG.1 The two brane compactified model, where y_1 is identified with y_0 , the position of the particle horizon. The circle has circumference $L_2 - L_0$. The brane at L_0 (L_1) is the Planck (visible) brane. The warp factor $A(y)$ is shown schematically.

We find that, for large brane separation L_1 , the warp factor $A(y)$ provides a plausible explanation of the hierarchy problem: why the electroweak scale m_{EW} is so much smaller than the Planck scale M_{Planck} ,

$$\frac{m_{EW}^2}{M_{Planck}^2} \simeq \frac{A(L_1)}{A(0)} \simeq e^{-\kappa^2(\sigma_0 - \sigma_1)L_1/3} \quad (3)$$

In this case, the effective 4-D cosmological constant Λ_{eff} is also exponentially small,

$$\Lambda_{eff} \simeq \frac{2\sigma_0(\sigma_0 + \sigma_1)}{(\sigma_0 - \sigma_1)} e^{-\kappa^2[(\sigma_0 + \sigma_1)(L_2 - L_1) + (\sigma_0 - \sigma_1)L_1]/6}. \quad (4)$$

where $H^2 = \kappa_N^2 \Lambda_{eff}/3$, and the 4-D gravitational coupling $\kappa_N^2 = 8\pi G_N = 8\pi M_{Planck}^2$ is given by

$$\frac{1}{2\kappa_N^2} = \frac{1}{2\kappa^2} \int A(y) dy, \quad (5)$$

With the above warp factor (3) and Λ_{eff} , the hierarchy problem and the cosmological constant problem may be simultaneously solved for appropriately large brane separations.

Note that the warp factor (3) is very similar to that in the Randall-Sundrum (RS) model [1]. In fact, it reduces to theirs if σ_1 is taken to be negative and $\sigma_1 = -\sigma_0$, in which case, we see that Λ_{eff} and H become zero independent of the brane separations, another property of the RS model. Although it is consistent to have a negative tension brane sitting at an orbifold fixed point if its fluctuating modes are removed by the orbifold projection, the visible brane in our model is not sitting at an orbifold fixed point, so stability requires its brane tension to be positive. Besides the above fine-tuning $\sigma_1 = -\sigma_0$, the RS model also requires a fine-tuning between the brane tension and the bulk cosmological constant. In Ref [4,5], the bulk cosmological constants are treated as integration constants which are determined by the 5-D Einstein equation. This is made possible with the introduction of 5-form field strengths and/or unimodular gravity.

We see that the metric factor $A(y_0) = 0$ at y_0 somewhere between the two branes. This is a particle horizon. The presence of the particle horizon is quite generic in this scenario for a non-zero H , independent of whether the 5th dimension is compactified or not. It is also present in scenarios with more than two branes. The particle horizon persists even when the branes are moving slowly. This feature is actually highly desirable because it evades Weinberg's argument [6] on the need of fine-tuning to get a very small cosmological constant. For our purpose, let us rephrase Weinberg's argument, which roughly goes as follows. Starting from any higher dimensional theory with the extra dimensions compactified, integrate out the extra dimensions to obtain an effective low energy 4-dimensional theory. Suppose the 4-D cosmological constant is classically exponentially small, or zero. Then 4-D quantum effects will introduce corrections to the 4-D cosmological constant that will not be exponentially small. This implies that an exponentially small (or zero) 4-D cosmological constant in our universe must be the result of a fine-tuning.

Now let us see how the warped geometry and the particle horizon evades this argument. For an observer on the visible brane at $y = L_1$, it takes infinite time for a light-like signal to travel from the brane to the particle horizon at y_0 :

$$\Delta t = \int_{L_1}^{y_{final}} dy / \sqrt{A(y)} \quad (6)$$

where Δt is clearly divergent when $y_{final} \rightarrow y_0$. (On the other hand, an observer travelling in the y direction will find that he/she takes only finite time to go full circle. See later.) So,

for an observer on the visible brane, the extra dimension seems to have infinite size, since a light-like signal he sends out in the positive y direction never finish going around the circle to come back to him in finite time. In this sense, the world does not look like 4-dimensional to the observer on the visible brane, even though the standard model fields, being trapped on the visible brane, essentially live in 4 dimensions, and gravity behaves like 4-dimensional due to the warped geometry [2]. So quantum corrections should be treated in the 5-D theory (or in a full string theory), where the quantities κ , σ_0 , σ_1 , L_1 and L_2 (that appear in the exponentially small Λ_{eff} (4) and the warp factor (3)) are 5-D renormalized quantities.

This non-4-dimensional feature can be seen in another way. Although the warp factor traps the massless graviton mode, so ordinary gravity behaves like 4-dimensional, this is not the case for the gravity KK modes. One may follow the approach of Ref [2] of writing the graviton mode equation in a conformally flat (or an almost conformally flat) metric, where we see that the particle horizon is mapped to $\pm\infty$. So one obtains a continuous KK spectrum, whose wavefunction is spread throughout the z co-ordinate, from minus infinity to plus infinity. In terms of the y co-ordinate, the continuous gravity KK modes have their wavefunctions peaked at the particle horizon. Even though the extra dimension is compactified, we see that the presence of the particle horizon reduces the gravity KK modes to a continuous spectrum, which usually implies an uncompactified direction. Strictly speaking, one cannot integrate out the extra dimension to obtain an effective 4-D theory. If one insists to treat the theory as a 4-D effective theory, one may employ the AdS/CFT correspondence [7] to obtain a strongly interacting conformal field theory [8]. As the 5-D bulk cosmological constant changes, this conformal field theory changes to another conformal field theory, a very unusual situation from the perspective of quantum corrections to Λ_{eff} [5]. For example, the 4-D quantum correction to the Newton's force law is purely a 5-D classical effect [9].

In the uncompactified scenario, we still get 4-D gravity due to the trapped graviton mode, Δt is again infinite, and the gravity KK spectrum is again continuous. (The latter two results follow more from the presence of particle horizons than from the uncompactified property. See Ref [5] for a discussion of the global spacetime structure.) The evasion of Weinberg's argument is again clear. In summary, although ordinary gravity and the standard model of strong and electroweak interactions all look 4-dimensional, the theory, even as a low energy effective field theory, is intrinsically non-4-dimensional in the sense discussed above. This evasion of Weinberg's argument is essential for the possibility of solving the cosmological constant problem in scenarios of this type.

Ref [4] also considers the case where the particle horizon is lifted (i.e., the minimum of the warp factor is positive). This can happen in a number of ways (for example, there is a kinetic energy term for a scalar mode). In this scenario, there will appear a mass gap in the KK spectrum in the compactified model. As long as the mass gap is small enough, the above argument should still apply. In the uncompactified scenario, the model remains a 5-D theory.

In models where the radion (brane separation) mode is stabilized, the radion mass may be in the electroweak scale and can show up as a giant resonance in high energy collisions [12]. This radion mode (or other particles) can also appear in the above scenario. Fortunately, they will be different from the gravity KK spectrum, since the latter will appear in high energy collisions as a tower of resonances with tensor couplings to matter fields. Angular

distribution measurements can easily distinguish them.

If the Planck brane is our universe (i.e., matter fields live on the Planck brane), we can still get an exponentially small cosmological constant, and physics is consistent with the radion mode remaining unstabilized. In this case, the hierarchy problem must be solved by other more conventional approaches, and the coupling of the KK modes to matter fields will be too weak to be seen in near future high energy colliders. Again, there are no giant resonances with tensor couplings in high energy collisions.

II. PHENOMENOLOGY

Let us first review briefly the phenomenology of the giant resonances in high energy colliders due to the presence of the gravity KK modes in the RS orbifold model [1,2]. Then we shall see that, in the presence of particle horizons, the gravity KK spectrum becomes continuous, and their couplings to ordinary matter becomes weak. A subtle issue about non-zero H that arises will be discussed later.

Here we have in mind that H is very small, and all measurements involving G_N are at distances much smaller than the cosmic size $1/H$. Instead of bringing the metric (1) into the conformally flat form, we can bring the metric (1) into an almost “conformally flat” form [5],

$$ds^2 = \frac{H^2}{k^2 \sinh^2 [H (|z| + z_0)]} [dz^2 - dt^2 + \exp(2Ht) \delta_{ij} dx^i dx^j] \quad (7)$$

where the value of k and $z_0 \simeq 1/k$ are in general different on the two sides of each brane. For the region that includes the particle horizon, z_0 is defined so that when $y = 0$ we also have $z = 0$. At the particle horizon, when $y \rightarrow y_0$, we will have $z \rightarrow \pm\infty$. We observe that the space between the horizons $(-y_0, +y_1)$ is mapped into the entire real line in the new z coordinate system. If the model contains branes separated by particle horizons, each interval that ends in particle horizons in the y coordinate will be mapped into the entire real line in the z coordinate, so we end up with a collection of “disconnected” spaces, each space containing a subset of branes. (See Ref [5] for a discussion on the global structure of the spacetime.)

The exponential factor $\exp(2Ht)$ in the metric (7) has no effect on time intervals and distances much smaller than the Hubble radius. To get an idea, we see that $\Lambda_{eff} \approx (10^{-3}eV)^4$ in our universe, so $H \approx 10^{-34}eV$, a totally negligible effect in collider physics, where the time scale is much smaller than a second. In the limit of $H = 0$, we recover the conformal metric of the RS model, that is, the particle horizon is pushed to $y = \pm\infty$.

Now, let us consider the graviton and the KK modes [2] in any two brane model where the hierarchy problem is solved with the warp factor. We are interested in the case where $k/M_{Planck} < 1$ and the KK mode mass $m \ll k$. The graviton mode is a bound state trapped around the Planck (hidden, σ_0) brane. The term in the 4-D effective Lagrangian responsible for the gravitational coupling to matter fields on the visible (TeV, σ_1) brane is given by:

$$L_{int} = -\frac{1}{M_{Planck}} T^{\mu\nu}(x) h_{\mu\nu}^{(m=0)}(x) - \frac{T^{\mu\nu}(x)}{m_{EW}^{3/2}} \sum_m \psi_m(z_1) h_{\mu\nu}^{(m)}(x) \quad (8)$$

where $h_{\mu\nu}^{(m=0)}(x)$ is the usual graviton field and $h_{\mu\nu}^{(m)}(x)$ is the gravity KK excitation field with mass m . Here, $T^{\mu\nu}(x)$ is the energy-momentum stress tensor of matter fields on the visible brane, which is sitting at z_1 , with canonical kinetic terms for the brane fields. Although one starts with typical Planck scale masses on both the Planck and the visible brane, the masses on the visible brane have absorbed a $\sqrt{A(L_1)}$ factor so the typical mass scale on the visible brane now becomes m_{EW} . Here $\psi_m(z_1)$ is the normalized wavefunction of the mass m gravity KK mode at the visible brane. Using (3) and the relation between z and y , we have $k_1 z_1 = M_{Planck}/m_{EW}$, or

$$\frac{1}{z_1} \approx \frac{\sqrt{\sigma_0 - \sigma_1}}{M_{Planck}} \frac{m_{EW}}{M_{Planck}} \quad (9)$$

In the RS orbifold model [1], the size of the orbifold is simply the brane separation z_1 . (In Figure 1, the orbifold corresponds to the region between L_0 and L_1 with Λ_1 in the bulk.) So the gravity KK modes are discretized, with mass $m \simeq n/z_1$, for integer n . In this model, $\sigma_1 = -\sigma_0$, so the lowest KK modes have masses $m = n/z_1 \simeq n\sqrt{\sigma_0}m_{EW}/M_{Planck}^2$. For relatively large z_1 , $|\psi_m(z_1)| \simeq 1/\sqrt{z_1}$ independent of m for the discrete KK spectrum. In this case, the effective coupling of the KK modes to the visible brane matter fields is

$$L_{KK} \simeq -\frac{\sigma_0^{1/4}}{M_{Planck}} \frac{T^{\mu\nu}(x)}{m_{EW}} \sum_{m=n/z_1} h_{\mu\nu}^{(m)}(x) \quad (10)$$

It is reasonable to take the Planck brane tension σ_0 to be comparable to M_{Planck}^4 , so the lowest resonance is around the electroweak scale and have electroweak strength coupling to matter fields. Since its coupling is stronger than the gravitational coupling by a factor of M_{Planck}/m_{EW} , giant resonances are expected in high energy (TeV scale) collision processes such as e^+e^- annihilations or Drell-Yan scatterings [11].

Now, let us consider the two brane compactified model (see Figure 1). In this scenario, the presence of the particle horizon implies that the effective size in the z coordinate (conformal metric) is infinite, so the KK spectrum is no longer dictated by the position of the visible brane (which is still at z_1), but by the infinite size in the z coordinate. The resulting KK spectrum is continuous. This implies that there is no distinct resonance signature in high energy collisions due to the gravity KK modes. Since $\sigma_0 > \sigma_1 > 0$, the discussion will be simplified if we take σ_1 to be negligibly small. The resulting phenomenology is similar to that of a probe brane in the presence of a Planck brane in the uncompactified case [3]. For each mass eigenvalue, there are a symmetric mode and an anti-symmetric mode (defined to have zero wavefunction at the Planck brane), and both sets of KK modes will contribute at the visible brane.

For simplicity, consider $\sigma_0 \approx M_{Planck}^4$. For KK modes with mass $H \lesssim m \lesssim m_{EW}^2/M_{Planck}$, $\psi_m(z_1) \approx -\sqrt{m/k_1^4 z_1^3}$. These modes have similar suppression factor as the trapped gravity mode and they contribute to the modification of the Newton's law [2]. For KK modes with mass $m_{EW} > m > m_{EW}^2/M_{Planck}$, $\psi_m(z_1) \approx (m/m_{EW})^{5/2}$. Using (8), we see that a typical cross-section behaves like [3]:

$$\sigma \approx E^6/m_{EW}^8 \quad (11)$$

For energies below m_{EW} , the cross-section is quite small. For energies above m_{EW} , we must include KK modes with mass $m > m_{EW}$. Their wavefunctions at the visible brane is $\psi_m(z_1) \approx 1$, so the scattering involving the gravity KK modes becomes strong at energies above m_{EW} . In the y coordinate, this means that the wavefunctions of these KK modes are peaked at the particle horizon. Note that there is no oscillating behavior of $|\psi_m(z)|^2$ that is shown in Ref [3] when both symmetric and antisymmetric modes are included.

In processes like $e^+e^- \rightarrow \text{photon} + KK\text{mode}$, unless the KK field bounces back from the Planck brane, it will be lost. Such missing energy events may provide a good signal of the scenario when the energy approaches m_{EW} .

III. THE GRAVITY MODES FOR FINITE H

Let us comment on the effect of a non-zero but very small H [13–15]. The setup consists of a Planck brane where all the mass scales are comparable to the Planck scale (and consequently, gravity is strong), and a visible brane which is treated as a probe brane, having little effect on the shape of the wavefunctions of the graviton along the 5th dimension. Consequently all the jump conditions will be imposed at the Planck brane. To solve for the gravity modes, we go to the almost conformally flat background metric (7). Following Ref [13], we decompose the fluctuations of the metric $ds^2 = (g_{ab} + \hat{h}_{ab}) dx^a dx^b$, into a wavefunction along the 5th dimension and a wavefunction in 4D deSitter space-time: $\hat{h}_{\mu\nu}(x^\rho, z) = A(z)^{-1/4} h_{\mu\nu}(x^\rho) \psi(z)$. The equations for $\psi(z)$ and $h_{\mu\nu}(x^\rho)$ are found to be:

$$-\partial_z^2 \psi + \frac{(A^{3/4})''}{A^{3/4}} \psi = \tilde{m}^2 \psi, \quad -\square h_{\mu\nu} + (2H^2 + \tilde{m}^2) h_{\mu\nu} = 0 \quad (12)$$

where \square indicates the 4D covariant d'Alembertian. We shall treat the Hubble constant H to be very small. Following Ref [2], and using the metric (7), we obtain the following equation for the gravity modes

$$-\partial_z^2 \psi + \left[\frac{15}{4} H^2 \coth^2(H(|z| + z_0)) - \frac{3H^2}{2} - 3H \coth(Hz_0) \delta(z) \right] \psi = \tilde{m}^2 \psi \quad (13)$$

This equation has a trapped mode with $\tilde{m} = 0$ and wavefunction

$$\psi_0(z) = \left(\frac{H}{k \sinh(H(z + z_0))} \right)^{3/2} \quad (14)$$

This is the graviton mode. Following (12), we see that the graviton mode has mass $m^2 = \tilde{m}^2 + 2H^2 = 2H^2$. We may solve Eq(13) in terms of hypergeometric functions (in the variable $\coth(H(|z| + z_0))$ or $\tanh(H(|z| + z_0))$). For our purpose here, Since we need only an estimate of the lower bound of the continuous KK spectrum, let us consider the two regimes: namely, $H|z| \ll 1$ and $H|z| \gg 1$. Since, for the visible brane, $z = z_1 \approx m_{EW}^{-1}$, let us first consider the regime where $H|z| \ll 1$. Using the expansion of the coth and sinh functions for small values of the argument, we obtain:

$$-\partial_z^2 \psi + \left[\frac{15}{4(|z| + z_0)^2} - 3 \frac{\delta(z)}{z_0} \left[1 + \frac{(Hz_0)^2}{3} \right] \right] \psi = \left(\tilde{m}^2 + \frac{3H^2}{2} \right) \psi = \bar{m}^2 \psi \quad (15)$$

which is valid for small $H|z|$. Here we may also drop the term $(Hz_0)^2/3$ since $H z_0 \ll H z_1$. This reduces to the original Randall-Sundrum equation except that the mass eigenvalue is replaced by $\overline{m}^2 = \tilde{m}^2 + 3H^2/2$. This yields the following solution:

$$\psi(z) = \sqrt{\overline{m}(|z| + z_0)} [a J_2(\overline{m}(|z| + z_0)) + b Y_2(\overline{m}(|z| + z_0))] \quad (16)$$

where $\overline{m}^2 = \tilde{m}^2 + 3H^2/2 \geq 0$. The solution must also satisfy the junction condition: $\psi'(0+) - \psi'(0-) = -3\psi(0)/z_0$. After properly normalized, we find that [2]

$$a = 1, \quad b = \pi \overline{m}^2 z_0^2 / 4 \quad (17)$$

so the continuous gravity KK spectrum is bounded below, by $m^2 \geq H^2/2$. On the visible brane, both symmetric and anti-symmetric modes will be present. For large $H|z|$, Eq(13) becomes

$$-\partial_z^2 \psi = \left(\tilde{m}^2 - \frac{9}{4} H^2 \right) \psi \quad (18)$$

which yields either a sinusoidal solution (for $\tilde{m}^2 > 9H^2/4$), or an exponential solution (for $\tilde{m}^2 < 9H^2/4$). The linear combination (that is, the coefficients a and b) of the wavefunction $\psi_m(z)$ in Eq(16) will change slightly. However, the basic physics is unchanged. We expect the KK spectrum to start at $m^2 \approx 2H^2$. Also the $H \rightarrow 0$ limit is expected to be smooth [15].

Due to the non-zero value of H , the Newton's law and the correction from the gravity KK modes are changed slightly. The gravitational potential for masses M_1 and M_2 on the visible brane will be:

$$V(r) = -G_N \frac{M_1 M_2}{r} \left(e^{-\sqrt{2} H r} + \frac{M_{Planck}^2}{\sigma_0} \int_{\alpha H}^{\infty} e^{-mr} m dm \right) \quad (19)$$

where α is of order 1. We see that the effects due to the non-zero H are negligible on distances much smaller than the Hubble radius.

IV. ADS PROPER TIME

Earlier, we point out that an observer travelling in the y direction will find that it takes only a finite amount of time to go full circle. To show this, we need to find the proper time for an observer moving along the 5th dimension towards the particle horizon located at $y = y_0$. We may use either the Killing vector approach, or equivalently, the geodesic approach. Let us follow the latter.

Since the observer moves along a timelike geodesic, we choose the proper time as the affine parameter. Only one connection coefficient, namely Γ_{tt}^y , is non-zero, so the equation of the geodesic becomes:

$$\frac{d^2 y}{d\tau^2} + \Gamma_{\mu\nu}^y \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \implies \frac{d^2 y}{d\tau^2} + \frac{1}{2} \frac{dA(y)}{dy} \left(\frac{dt}{d\tau} \right)^2 = 0 \quad (20)$$

Using the metric (1) where $A(y) = H^2 \sinh^2[k(y - y_0)]/k^2$ and the fact that $x^1, x^2, x^3 = \text{constant}$ we obtain:

$$\left(\frac{dy}{d\tau}\right)^2 - A(y) \left(\frac{dt}{d\tau}\right)^2 = -1 \quad (21)$$

since in the reference frame moving with the observer, $ds^2 = -d\tau^2$. We can reduce the second-order differential equation to a first-order one using the standard method:

$$\frac{d^2y}{d\tau^2} = \frac{d}{d\tau} \left(\frac{dy}{d\tau}\right) = \frac{dy}{d\tau} \frac{d}{dy} \left(\frac{dy}{d\tau}\right) = \frac{1}{2} \frac{d}{dy} \left(\frac{dy}{d\tau}\right)^2 = \frac{1}{2} \frac{d(y')^2}{dy} \quad (22)$$

Multiplying Eq.(20) by $A(y)$ and substituting $A(y) (dt/d\tau)^2$ from Eq.(21) we obtain:

$$A(y) \frac{d(y')^2}{dy} + \frac{dA(y)}{dy} [1 + (y')^2] = 0 \implies A(y) [1 + (y')^2] = E^2 \quad (23)$$

where the constant E is given by the initial energy (velocity) of the observer, so the proper time it takes an observer to reach the particle horizon at y_0 (where $A(y_0) = 0$) is:

$$\tau = \int_0^{y_0} \sqrt{\frac{A(y)}{E^2 - A(y)}} dy \quad (24)$$

which is finite. This means a traveller around the compactified circle will find he/she takes only a finite time to go full circle. Compared to the observer staying on the brane, this is the ultimate twin paradox.

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